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(5) 1) $f''(x) = \frac{3}{2}x - \frac{9}{2} \Rightarrow f'(x) = \frac{3}{4}x^2 - \frac{9}{2}x + a \Rightarrow f(x) = \frac{3}{12}x^3 - \frac{9}{4}x^2 + ax + b$

- $P(0|\frac{5}{4}) \in G_f \Rightarrow f(0) = \frac{5}{4} \Rightarrow b = \frac{5}{4}$
- $N(-1|0) \in G_f \Rightarrow f(-1) = 0 \Rightarrow -\frac{3}{12} - \frac{9}{4} - a + \frac{5}{4} = 0 \Leftrightarrow a = -\frac{5}{4}$
- $f(x) = \frac{3}{12}x^3 - \frac{9}{4}x^2 - \frac{5}{4}x + \frac{5}{4} = \frac{1}{4}(x^3 - 9x^2 - 5x + 5)$

(3) 2.1 $g_a(x) = \frac{1}{4}(x^3 - 10x^2 + ax + x^2 - 10x + a) = \frac{1}{4}(x^3 - 9x^2 - 10x + ax + a)$

- $g_5(x) = \frac{1}{4}(x^3 - 9x^2 - 10x + 5x + 5) = \frac{1}{4}(x^3 - 9x^2 - 5x + 5) = f(x)$

(6) 2.2 $g_a'(x) = \frac{1}{4}(3x^2 - 18x - 10 + a) = 0$

- $D = 18^2 - 4 \cdot 3(a - 10) = 324 + 120 - 12a = 444 - 12a$
- $444 - 12a \leq 0 \Leftrightarrow -12a \leq 444 \Leftrightarrow a \geq 37$

(3) 2.3 $g_{25}(x) = \frac{1}{4}(x+1)(x - 10x + 25) = \frac{1}{4}(x+1)(x-5)^2$ (vgl. 2.0)

$x_1 = -1$; $x_2 = 5$

(6) 2.4 $g_{25}(x) = \frac{1}{4}(x^3 - 9x^2 - 10x + 25x + 25) = \frac{1}{4}(x^3 - 9x^2 + 15x + 25)$

- $g_{25}'(x) = \frac{1}{4}(3x^2 - 18x + 15) = \frac{3}{4}(x^2 - 6x + 5) = \frac{3}{4}(x-1)(x-5) = 0$
- oder $x_{1/2} = \frac{1}{2}(6 \pm \sqrt{36 - 4 \cdot 1 \cdot 5}) = \frac{1}{2}(6 \pm 4)$ $x_1 = 1$ $x_2 = 5$

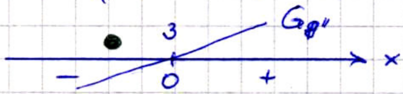
VZ f' + 0 − 0 +

G_f sms HOP smf TIP sms

$f(1) = 8 \Rightarrow$ HOP $(1/8)$

$f(5) = 0 \Rightarrow$ TIP $(5/0)$

(4) 2.5 $g_{25}''(x) = \frac{3}{4}(2x - 6) = 0 \Leftrightarrow x_w = 3$



$f(3) = 4 \Rightarrow$ WEP $(3/4)$

G_g rekr. WEP likr

rekr. in $]-\infty; 3]$

likr in $[3; \infty[$

(5) 2.6 G_f 5BE

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3.1 (7)

$p(x) = \frac{1}{4}(x-5)^2 - (x-5) = \frac{1}{4}x^2 - \frac{5}{2}x + \frac{25}{4} - x + 5 = \frac{1}{4}x^2 - \frac{7}{2}x + \frac{45}{4}$

$\frac{1}{4}(x^3 - 9x^2 + 15x + 25) = \frac{1}{4}(x^2 - 14x + 45) \mid \cdot 4 \qquad = \frac{1}{4}(x^2 - 14x + 45)$

$\Leftrightarrow x^3 - 10x^2 + 29x - 20 = 0 \quad ; \quad x_1 = 1 \; ; \; x_2 = 4 \; ; \; x_3 = 5$

2,5 + 1,5

$(x^3 - 10x^2 + 29x - 20) : (x-5) = x^2 - 5x + 4 = (x-1)(x-4)$

$-(x^3 - 5x^2)$

$\quad -5x^2$

$\quad -(-5x^2 + 25x)$

$\qquad 4x - 20$

$\qquad -(4x - 20)$

$x_1 = 5 \rightsquigarrow S_1(5|0)$ (TIP)

$x_2 = 4 \rightarrow S_2(4|1{,}25)$

$x_3 = 1 \rightsquigarrow S_3(1|8)$ (HOP)

3.3 (5)

$I = \int_1^4 (g_{25}(x) - p(x))\,dx = \int_1^4 \frac{1}{4}(x^3 - 10x^2 + 29x - 20)\,dx$

$F(x) = \frac{1}{4}\left(\frac{1}{4}x^4 - \frac{10}{3}x^3 + \frac{29}{2}x^2 - 20x\right) \left(= \frac{1}{16}x^4 - \frac{10}{12}x^3 + \frac{29}{8}x^2 - 4x\right)$

$F(4) = \frac{1}{4}\left(\frac{1}{4}\cdot 4^4 - \frac{10}{3}\cdot 4^3 + \frac{29}{2}\cdot 4^2 - 20\cdot 4\right) = \frac{1}{4}\cdot\frac{8}{3} = \frac{2}{3}$

$F(1) = \frac{1}{4}\left(\frac{1}{4} - \frac{10}{3} + \frac{29}{2} - 20\right) = \frac{1}{4}\left(-\frac{103}{12}\right) = -\frac{103}{48}$

$I = F(4) - F(1) = \frac{2}{3} + \frac{103}{48} = \frac{45}{16} = 2{,}8125$ [FE]

4 (4)

$h'(x) = \begin{cases} \frac{1}{4}(3x^2 - 18x + 15) & \text{f. } x < 5 \\ \frac{1}{2}x - \frac{7}{2} & \text{f. } x > 5 \end{cases}$

$\lim_{x\to 5^-} h'(x) = \lim_{x\to 5^-}\left(\frac{1}{4}(3x^2 - 18x + 15)\right) = 0$ (TIP)

$\lim_{x\to 5^+} h'(x) = \lim_{x\to 5^+}\left(\frac{1}{2}x - \frac{7}{2}\right) = -1$

ungleich, also nicht diffbar

5.1 (4)

$V = G\cdot h\ (*)$; $G = r^2\pi$;

Oberfl.: $O = 2\cdot G + M = 2\cdot r^2\pi + \overbrace{2r\pi}^{\text{Umf.}} h = 2400\pi$

$\Leftrightarrow h = \frac{2400\pi - 2r^2\pi}{2r\pi} = \frac{1200}{r} - r$ in $(*)$ $V(r) = r^2\pi\left(\frac{1200}{r} - r\right)$

$V(r) = \pi(1200r - r^3)$

5.2 (5)

$V'(r) = \pi(1200 - 3r^2) = 0 \Leftrightarrow r^2 = \frac{1200}{3} = 400 \Rightarrow r_{1(2)} = +(-)\,20$

$V(1{,}2) \approx 39{,}8$; $V(20) \approx 50{,}2$; $V(30) \approx 28{,}3$ [dm^3] $\Rightarrow V_{max}$ für $r_{max} = 20$